Hautlieu Mathematics

Summer Task

1. To be successful in Mathematics, students need to be confident in certain aspects of algebra, coordinate geometry and trigonometry before starting the course. All students wishing to study A level further Mathematics in year 12 are required to have completed this pack before joining the course.

2. Students need to be able to write their solutions clearly and in full, with the work well set out. Work should be written on A4 lined paper and your name written at the top. **Full working must be shown**, with each line of working below the previous one, that is, you need to work down the page. Question numbers should be put in the margin. **Only one column is to be used on a page**. Failure to show full working will result in your being asked to redo the pack.

3. Answers are given at the end and **students must MARK their work before it is handed in**. Marking should be done clearly in RED. Any questions that are not initially solved correctly should be retried until the correct answer is obtained. If you cannot obtain the correct answer, after retrying the question, a red circle should be put around that question number. Work that is not marked in this way will not be accepted.

4. **The topics covered in this pack will be considered assumed knowledge when the course begins**. If there are topics in this pack that you are uncertain of then you should visit the BBC Bitesize website (http://www.bbc.co.uk/education) or use YouTube channels (eg. Khan Academy, DrFrostMaths, corbettmaths) to find a video lesson.

5. Any questions please email : p.pattinson@hautlieu.sch.je (Head of Maths)

The rules of indices

Exercise 1 – Simplify these expressions	
1. $(p^{3})^{2} \div p^{4}$	2. $9x^{2} \times 3(x^{2})^{3}$
3. $7a^{4} \times (3a^{4})^{2}$	4. $(4y^{3})^{3} \div 2y^{3}$

Expanding brackets

Exercise 2 – expand and simplify

1. $4x(x + 3) - 2(3x - 7)$	2. $3x^{2}(2x + 1) - 5x^{2}(3x - 4)$
3. $(a - b + c)(2a + b - 2c)$	4. $(2x^{2} + x + 2)(x^{2} - 3x - 2)$

Factorising expressions

Exercise 3 – factorise completely

1. $x^{2} + 14x - 51$	2. $2x^{2} - 5x + 3$
3. $(x + 3)^{2} - 7(x + 3)$	4. $8x^{2} + 19x + 6$

Solving quadratics

Exercise 4 – solve using factorising. Some will need to be rearranged first.	
1) $2x^{2} + 5x + 2 = 0$	2) $3x^{2} + 10x - 8 = 0$
3) $5x^{2} - 16x + 3 = 0$	4) $6x^{2} = 8(x + 1)$
5) $x^{2} - 4 = 0$	6) $4x^{2} - 25 = 0$

<u>Using the Quadratic Formula</u>

Exercise 5 – solve using the Quadratic Formula (give your answers to 3sf)	
1) $6x^{2} - 10x - 1 = 0$	2) $7x + 9 - 6x^{2} = 0$

Completing the square

Exercise 5a – Convert these expressions to completed square format	
1) $x^2 + \frac{2}{3}x + 1$	2) $2x^2 - 8x - 7$
3) $1 + 10x - x^2$	4) $4 - 2x - 3x^2$

Extending the rules for indices

Exercise 6 - Evaluate without using a calculator

1. $49^{\frac{3}{2}}$	2. $25^{-\frac{1}{2}}$
3. $\left(1\frac{9}{16}\right)^{\frac{3}{2}}$	4. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Using surds and irrational numbers

Exercise 7 - simplify	
1. $\sqrt{200} + \sqrt{18} - 4\sqrt{72}$	2. $(3\sqrt{2} + 2\sqrt{3})^2$
3. Find the value of $x^2 + 4x + 4$ when $x = 2 + \sqrt{3}$	

Rationalising the denominator

Examples

a. $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{\sqrt{3}}{3}$	b. $\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$ $= \frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})}$ $= \frac{3-\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2}$ $= \frac{3-\sqrt{2}}{7}$	c. $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ $= \frac{5+\sqrt{5}\sqrt{2}+\sqrt{2}\sqrt{5}+2}{5-2}$ $= \frac{7+2\sqrt{10}}{3}$

Exercise 8 – rationalise the denominators	
1. $\frac{1}{\sqrt{2}}$	2. $\frac{1}{2+\sqrt{5}}$
3. $\frac{1}{\sqrt{5}-\sqrt{3}}$	4. $\frac{3-\sqrt{2}}{4-\sqrt{5}}$

Changing the subject of a formula

Exercise 9 – Make y the subject.	
1. $\frac{a-y}{a+y} = b$	2. $\frac{ay+x}{x} = 4-y$
3. $\sqrt{\frac{y+x}{y-x}} = 2$	4. $\sqrt{\frac{m(y+n)}{y}} = p$

Inequalities

Exercise 10 – Solve these inequalities	
1. $3x+1 < 2x+5$	2. $3(x-1) < 2(1-x)$
3. $-\frac{2x}{5} > 3$	4. $1 < x+2 < 6$

Algebraic fractions

Exercise 11 – Simplify as far as possible	
1. $\dfrac{x^2+2x}{x^2-3x}$	2. $\dfrac{x^2-4x-21}{x^2-5x-14}$
3. $\dfrac{3x+1}{2}+\dfrac{x-2}{5}$	4. $\dfrac{3x+1}{2}-\dfrac{x-2}{5}$
5. $\dfrac{3x+1}{2}\times\dfrac{x-2}{5}$	6. $\dfrac{x-5}{10}\times\dfrac{5}{x^2-5x}$

Simultaneous Equations

Exercise 12 – Solve to find both x and y.	
1. $2x+5y=24$ $4x+3y=20$	2. $3x-y=5$ $2x+5y=9$
3. $xy=-4$ $2y=x+6$	4. $x^2+y^2=13$ $x+y=1$

Coordinate Geometry

Exercise 13 –Straight Lines

1. Here are the equations of several straight lines.

A	$y = -x + 7$	B	$2y = x - 7$	C	$4x + 12y = 5$
D	$x + 3y = 10$	E	$y = x - 3$	F	$y = 5 - 5x$
G	$5x + y = 12$	H	$y = 3 - 2x$	I	$y = 2x + 4$

a Find **two** pairs of lines which are parallel.
b Find **two** pairs of lines which are perpendicular.
c Find **one** line which is neither parallel nor perpendicular to any of the other lines.

2.

The diagram shows three points $A(-1, 5)$, $B(2, -1)$ and $C(0, 5)$.
The line **L** is parallel to AB and passes through C.

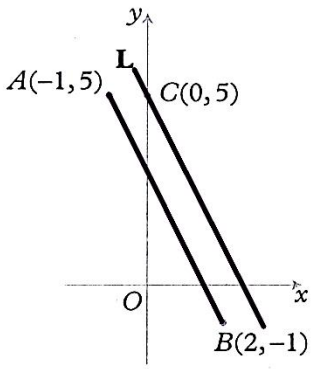


Diagram **NOT** accurately drawn

Find the equation of the line **L**.

3.

Find the exact length of the line segment joining each pair of points, giving your answers in terms of surds where appropriate.

a (1, 1) and (4, 5)

4.

Find the coordinates of the mid-point of the line segment joining each pair of points.

a (0, 2) and (8, 4)

5.

Find, in the form $ax + by + c = 0$, where a, b and c are integers, the equation of the straight line which passes through each pair of points.

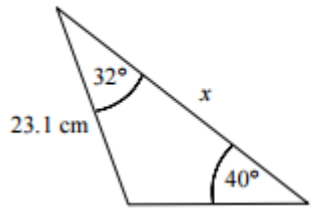
a (3, 0) and (5, 2)

Exercise 14 –Trigonometry

1.

Find the length x in each triangle.

a



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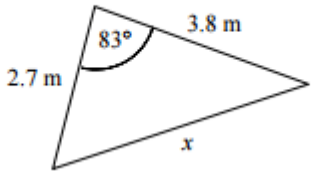

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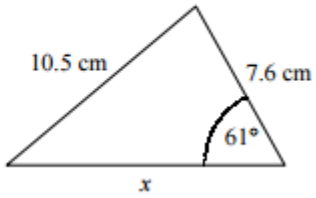
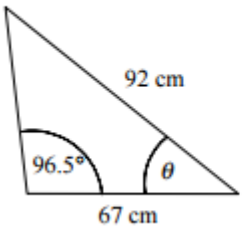


2.

Find the angle θ in each triangle.

a



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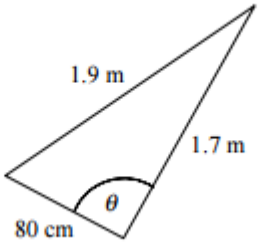

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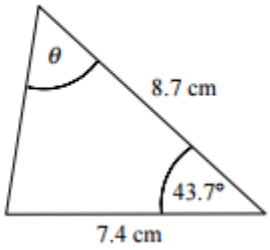
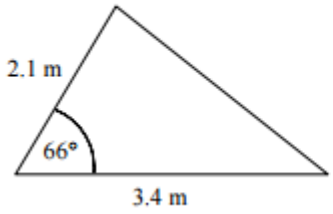


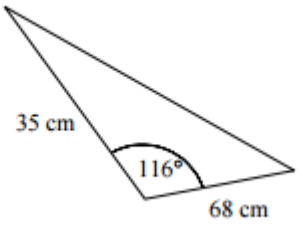
3.

Find the area of each of the following triangles.

a



b



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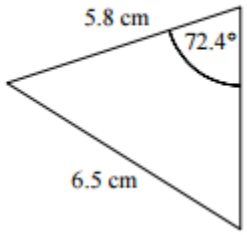


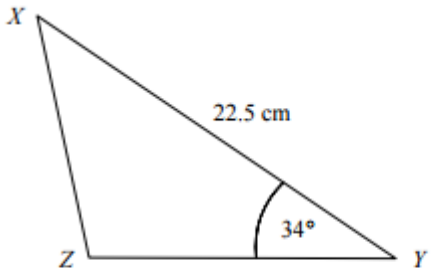
4.

Joanne walks 4.2 miles on a bearing of 138°. She then walks 7.8 miles on a bearing of 251°.

a Calculate how far Joanne is from the point where she started.

b Find, as a bearing, the direction in which Joanne would have to walk in order to return to the point where she started.

5.



The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.

Given that the area of the triangle is 100 cm^2, find the length XZ.

End of summer assessment.

Hautlieu Summer Task Solutions

Exercise 1		Exercise 2		Exercise 3		Exercise 4	
1	p^2	1	$4x^2 + 6x + 14$	1	$(x+17)(x-3)$	1	-2, -1/2
2	$27x^8$	2	$-9x^3 + 23x^2$	2	$(2x-3)(x-1)$	2	-4, 2/3
3	$63a^{12}$	3	$2a^2 - b^2 - 2c^2 - ab + 3bc$	3	$(x+3)(x-4)$	3	1/5, 3
4	$32y^6$	4	$2x^4 - 5x^3 - 5x^2 - 8x - 4$	4	$(8x+3)(x+2)$	4	-2/3, 2
						5	± 2
						6	- 5/2, 5/2

Exercise 5		Exercise 5a				Exercise 6			
1	-1.76, 0.0946	1	$\left(x+\frac{1}{3}\right)^2 + \frac{8}{9}$	3	$-(x-5)^2 + 26$	1	343	3	$\frac{125}{64}$
2	-0.773, 1.94	2	$2(x-2)^2 - 15$	4	$-3\left(x+\frac{1}{3}\right)^2 + \frac{13}{3}$	2	$\frac{1}{5}$	4	$\frac{4}{9}$

Exercise 7				Exercise 8			
1	$-11\sqrt{2}$	3	$19 + 8\sqrt{3}$	1	$\frac{\sqrt{2}}{2}$	3	$\frac{\sqrt{5}+\sqrt{3}}{2}$
2	$30 + 12\sqrt{6}$			2	$\sqrt{5} - 2$	4	$\frac{12+3\sqrt{5}-4\sqrt{2}-\sqrt{10}}{11}$

Exercise 9	**3.** $y = (a-ab)/(b+1)$ **2.** $3x/(a+x)$ **3.** $5x/3$ **4.** $mn/(p^2-m)$
Exercise 10	**1.** $x < 4$ **2.** $x < 1$ **3.** $x < 7.5$ **4.** $-1 < x < 4$
Exercise 11	**1.** $(x+2)/(x-3)$ **2.** $(x+3)/(x+2)$ **3.** $(17x+1)/10$ **4.** $(13x+9)/10$ **5.** $(3x^2-5x-2)/10$ **6.** $1/2x$
Exercise 12	**1.** $x = 2, y = 4$ **2.** $x = 2, y = 1$ **3.** (-4, 1) and (-2,2) **4.** $(3,-2)$ *and* $(-2,3)$
Exercise 13	

	1. a: $F \& G;\ C \& D$ **b:** $B \& H;\ A \& E$ **c:** I **2.** $y = -2x + 5$ **3. a:** 5 **4. a:** (4, 3) **5. a:** $x - y - 3 = 0$
Exercise 14	**1 a:** 34.2cm **b:** 4.39m **c:** 11.8cm **2 a:** 37.1° **b:** 91.7° **c:** 56.8° **3 a:** $3.26m^2$ **b:** $1070cm^2$ **c:** $14.3cm^2$ **4 a:** 7.27miles **b:** 039 **5** 12.9cm