



Hautlieu Mathematics

Summer Task

1. To be successful in Mathematics, students need to be confident in certain aspects of algebra, coordinate geometry and trigonometry before starting the course. All students wishing to study Analysis and Approaches Higher Level in year 12 are required to have completed both summer tasks before joining the course.
2. Students need to be able to write their solutions clearly and in full, with the work well set out. Work should be written on A4 lined paper and your name written at the top. **Full working must be shown**, with each line of algebra below the previous one, that is, you need to work down the page. Question numbers should be put in the margin. **Only one column is to be used on a page**. Failure to show full working will result in your being asked to redo the pack before you can start the course.
3. Answers are given at the end and **students must MARK their work before it is handed in**. This should be done clearly in **RED**. Any questions that are not initially solved correctly should be retried until the correct answer is obtained. If you cannot obtain the correct answer, after retrying the question, a red circle should be put around that question number. Work that is not marked in this way will not be accepted.
4. **The topics covered in this pack will be considered assumed knowledge when the course begins**. If there are topics in this pack that you are uncertain of then you could use YouTube to find a video lesson (eg. Khan Academy, DrFrostMaths, corbettmaths) or use an internet search engine to find other resources.
5. Any questions please email : p.pattinson@hautlieu.sch.je (Head of Maths)

Surds & Indices**Exercise I**

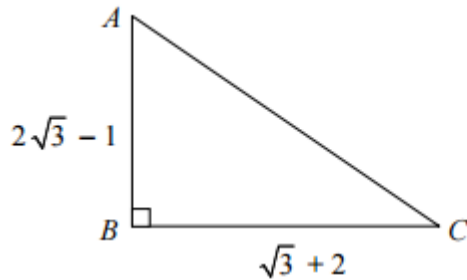
1.

Show that

$$\frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}}$$

can be written in the form $k\sqrt{7}$, where k is an integer to be found.

2.



In triangle ABC , $AB = 2\sqrt{3} - 1$, $BC = \sqrt{3} + 2$ and $\angle ABC = 90^\circ$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC = 2\sqrt{5}$.

c Show that $\tan(\angle ACB) = 5\sqrt{3} - 8$.

3.

Given that the point with coordinates $(1 + \sqrt{3}, 5\sqrt{3})$ lies on the curve with the equation

$$y = 2x^2 + px + q,$$

find the values of the rational constants p and q .

4.

Solve the equation

$$16^{x+1} = 8^{2x+1}.$$

5. Given that $a^{\frac{1}{3}} = b^{\frac{3}{4}}$ and that $a > 0$ and $b > 0$,

a. find an expression for $a^{\frac{1}{2}}$ in terms of b

b. find an expression for $b^{\frac{1}{2}}$ in terms of a

6.

a Given that $y = 2^x$, express each of the following in terms of y .

i 2^{x+2}

ii 4^x

b Hence, or otherwise, find the value of x for which

$$4^x - 2^{x+2} = 0.$$

Inequalities

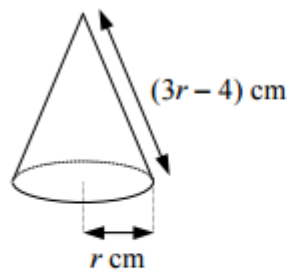
Exercise 2

1.

Given that the equation $2x(x + 1) = kx - 8$ has real and distinct roots,

- show that $k^2 - 4k - 60 > 0$,
- find the set of possible values of k .

2.



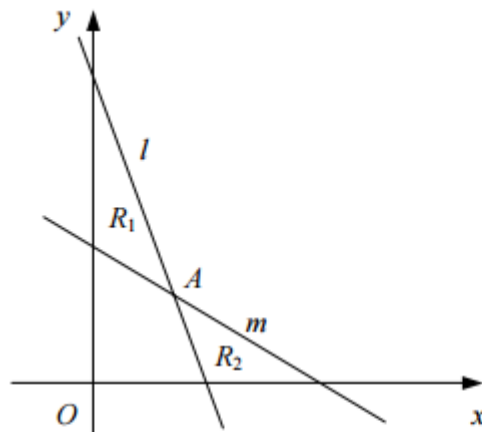
A party hat is designed in the shape of a right circular cone of base radius r cm and slant height $(3r - 4)$ cm.

Given that the height of the cone must not be more than 24 cm, find the maximum value of r .

Coordinate Geometry

Exercise 3

1.



The line l with equation $3x + y - 9 = 0$ intersects the line m with equation $2x + 3y - 12 = 0$ at the point A as shown in the diagram above.

- Find, as exact fractions, the coordinates of the point A .

The region R_1 is bounded by l , m and the y -axis.

The region R_2 is bounded by l , m and the x -axis.

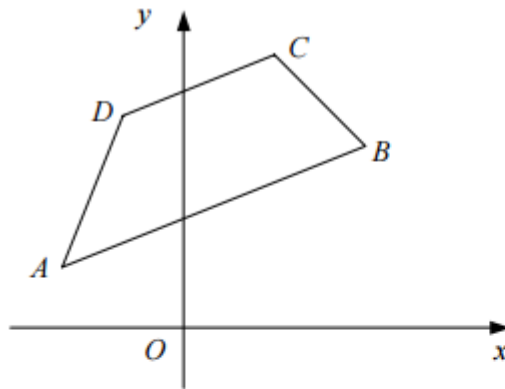
- Show that the ratio of the area of R_1 to the area of R_2 is $25 : 18$

2.

The point A has coordinates $(-8, 1)$ and the point B has coordinates $(-4, -5)$.

- Find the equation of the straight line passing through A and B , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.
- Show that the distance of the mid-point of AB from the origin is $k\sqrt{10}$ where k is an integer to be found.

3.



The diagram shows trapezium $ABCD$ in which sides AB and DC are parallel. The point A has coordinates $(-4, 2)$ and the point B has coordinates $(6, 6)$.

- a** Find the equation of the straight line passing through A and B , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Given that the gradient of BC is -1 ,

- b** find an equation of the straight line passing through B and C .

Given also that the point D has coordinates $(-2, 7)$,

- c** find the coordinates of the point C ,
d show that $\angle ACB = 90^\circ$.

4.

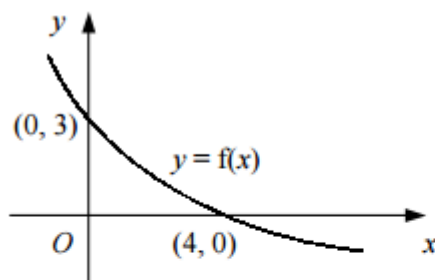
The straight line l passes through the points $A(1, 2\sqrt{3})$ and $B(\sqrt{3}, 6)$.

- a** Find the gradient of l in its simplest form.
b Show that l also passes through the origin.
c Show that the straight line which passes through A and is perpendicular to l has equation

$$x + 2\sqrt{3}y - 13 = 0.$$

Graph Transformations**Exercise 4**

1.



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points $(0, 3)$ and $(4, 0)$.

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = 3f(x)$ **b** $y = f(x + 4)$ **c** $y = -f(x)$ **d** $y = f\left(\frac{1}{2}x\right)$

2.

The curve $y = f(x)$ is a parabola and the coordinates of its turning point are (a, b) .

Write down, in terms of a and b , the coordinates of the turning point of the graph

a $y = 3f(x)$ **b** $y = 4 + f(x)$ **c** $y = f(x + 1)$ **d** $y = f\left(\frac{1}{3}x\right)$

Trigonometry**Exercise 5**

1.

a Solve the equation

$$2y^2 - 3y + 1 = 0.$$

b Hence, find the values of x in the interval $0 \leq x \leq 360^\circ$ for which

$$2 \sin^2 x - 3 \sin x + 1 = 0.$$

2.

Solve each equation for x in the interval $0 \leq x \leq 360$.

Give your answers to 1 decimal place where appropriate.

a $\sin(x - 60)^\circ = 0.5$ **b** $\tan(x + 30)^\circ = 1$ **c** $\cos(x - 45)^\circ = 0.2$
d $\tan(x + 30)^\circ = 0.78$ **e** $\cos(x + 45)^\circ = -0.5$ **f** $\sin(x - 60)^\circ = -0.89$

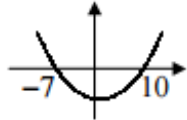


Summer Task Answers

Exercise 1

1	$-2\sqrt{7}$, ie $k = -2$	2	a) $\frac{4+3\sqrt{3}}{2}$ b) $2\sqrt{5}$ c) $\tan(\angle ACB) = \frac{2\sqrt{3}-1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{(2\sqrt{3}-1)(\sqrt{3}-2)}{3-4}$ $= -(2\sqrt{3}-1)(\sqrt{3}-2)$ $= -(6-4\sqrt{3}-\sqrt{3}+2)$ $= -(8-5\sqrt{3}) = 5\sqrt{3}-8$	3	$p = 1$ $q = -9$	4	$x = \frac{1}{2}$	5	a) $a^{\frac{1}{2}} = b^{\frac{9}{8}}$ b) $b^{\frac{1}{2}} = a^{\frac{2}{9}}$	6	a) (i) $4y$ (ii) y^2 b) $2^x = 0$, ie no solutions or $2^x = 4$, ie $x = 2$
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Exercise 2

1	a) $2x^2 + 2x - kx + 8 = 0$ real and distinct roots $\therefore b^2 - 4ac > 0$ $(2-k)^2 - 64 > 0$ $4 - 4k + k^2 - 64 > 0$ $k^2 - 4k - 60 > 0$ b) $(k+6)(k-10) > 0$ $k < -6$ or $k > 10$	2	let height be $h \therefore h^2 = (3r-4)^2 - r^2$ but $h \leq 24$ $\therefore h^2 \leq 24^2$ $(3r-4)^2 - r^2 \leq 576$ $r^2 - 3r - 70 \leq 0$ $(r+7)(r-10) \leq 0$ $-7 \leq r \leq 10$ \therefore maximum value of $r = 10$ 
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Exercise 3

1.

a $l \Rightarrow 9x + 3y - 27 = 0$

subtracting, $7x - 15 = 0$

$$x = \frac{15}{7}$$

$$\therefore A\left(\frac{15}{7}, \frac{18}{7}\right)$$

b l meets y -axis: $x = 0 \Rightarrow y = 9$

m meets y -axis: $x = 0 \Rightarrow y = 4$

$$\text{area of } R_1 = \frac{1}{2} \times 5 \times \frac{15}{7} = \frac{75}{14}$$

l meets x -axis: $y = 0 \Rightarrow x = 3$

m meets x -axis: $y = 0 \Rightarrow x = 6$

$$\text{area of } R_2 = \frac{1}{2} \times 3 \times \frac{18}{7} = \frac{54}{14}$$

$$\text{area } R_1 : \text{area of } R_2 = \frac{75}{14} : \frac{54}{14} = 25 : 18$$

2.

a $\text{grad} = \frac{-5-1}{-4+8} = -\frac{3}{2}$

$$\therefore y - 1 = -\frac{3}{2}(x + 8)$$

$$2y - 2 = -3x - 24$$

$$3x + 2y + 22 = 0$$

b mid-point = $\left(\frac{-8-4}{2}, \frac{1-5}{2}\right) = (-6, -2)$

$$\text{distance} = \sqrt{6^2 + 2^2} = \sqrt{40}$$

$$= 2\sqrt{10} \quad [k = 2]$$

3.

$$\text{a } \text{grad} = \frac{6-2}{6+4} = \frac{2}{5}$$

$$\therefore y - 2 = \frac{2}{5}(x + 4)$$

$$5y - 10 = 2x + 8$$

$$2x - 5y + 18 = 0$$

$$\text{b } y - 6 = -(x - 6) \quad [y = 12 - x]$$

$$\text{c } \text{grad } DC = \text{grad } AB = \frac{2}{5}$$

$$\therefore \text{eqn } DC \text{ is } y - 7 = \frac{2}{5}(x + 2)$$

$$y = \frac{2}{5}x + 7\frac{4}{5}$$

$$\text{at } C: 12 - x = \frac{2}{5}x + 7\frac{4}{5}$$

$$60 - 5x = 2x + 39$$

$$x = 3$$

$$\therefore C(3, 9)$$

$$\text{d } \text{grad } AC = \frac{9-2}{3+4} = 1$$

$$\text{grad } AC \times \text{grad } BC = 1 \times -1 = -1$$

$$\therefore AC \text{ is perpendicular to } BC$$

$$\therefore \angle ACB = 90^\circ$$

4.

$$\begin{aligned} \text{a } \text{grad} &= \frac{6-2\sqrt{3}}{\sqrt{3}-1} = \frac{6-2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\ &= \frac{6\sqrt{3}+6-6-2\sqrt{3}}{3-1} = \frac{4\sqrt{3}}{2} \\ &= 2\sqrt{3} \end{aligned}$$

$$\text{b } l: y - 2\sqrt{3} = 2\sqrt{3}(x - 1)$$

$$y = 2\sqrt{3}x$$

$$\text{when } x = 0, y = 0$$

$$\therefore \text{passes through origin}$$

$$\text{c } \text{perp grad} = -\frac{1}{2\sqrt{3}}$$

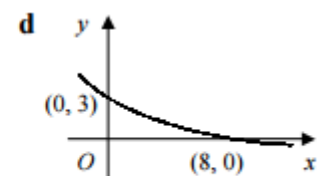
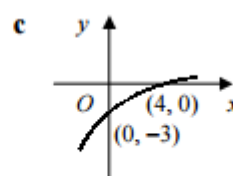
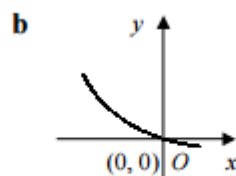
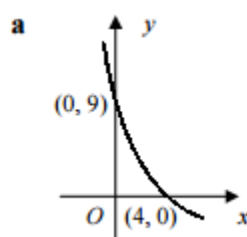
$$\therefore y - 2\sqrt{3} = -\frac{1}{2\sqrt{3}}(x - 1)$$

$$2\sqrt{3}y - 12 = -x + 1$$

$$x + 2\sqrt{3}y - 13 = 0$$

Exercise 4

1.



2.

a $(a, 3b)$

b $(a, b + 4)$

c $(a - 1, b)$

d $(3a, b)$

Exercise 5

1	<p>a) $y = \frac{1}{2}$ or 1</p> <p>b) $x = 30^\circ, 90^\circ, 150^\circ$</p>	2	<p>a) $x = 90^\circ, 210^\circ$</p> <p>b) $x = 15^\circ, 195^\circ$</p> <p>c) $x = 123.5^\circ, 326.5^\circ$</p> <p>d) $x = 8.0^\circ, 188.0^\circ$</p> <p>e) $x = 75^\circ, 95^\circ$</p> <p>f) $x = 302.9^\circ, 357.1^\circ$</p>
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